

QUANTUM HERMITE–HADAMARD INEQUALITIES FOR DOUBLE INTEGRAL AND q -DIFFERENTIABLE CONVEX FUNCTIONS

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Abstract. In this paper, we establish some new quantum analogue of Hermite-Hadamard inequalities for double integral and refinements of Hermite-Hadamard inequality for q -differentiable convex functions.

Mathematics subject classification (2010): 26D10, 26D15, 26A51.

Keywords and phrases: Hermite-Hadamard inequalities, q -derivative, q -integral, convex functions.

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