Journal of Mathematical Inequalities
Volume 13, Number 3 (2019), 747–752 doi:10.7153/jmi-2019-13-51

A NOTE ON "REMARKS ON SOME INEQUALITIES FOR POSITIVE SEMIDEFINITE MATRICES AND QUESTIONS FOR BOURIN"

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Abstract. Let A_i, $B_i \in \mathscr{M}_n$ be positive semidefinite matrices with $A_iB_i = B_iA_i$ $(i = 1,2,\cdots,m)$. Then

$$\sigma\Big(\Big(\sum_{i=1}^{m}(A_iB_i)^{\frac{1}{2}}\Big)^{r}\Big) \prec_{wlog} \sigma\Big(\Big(\sum_{i=1}^{m}A_i\Big)^{\frac{r}{4}}\Big(\sum_{i=1}^{m}B_i\Big)^{\frac{r}{2}}\Big(\sum_{i=1}^{m}A_i\Big)^{\frac{r}{4}}\Big),$$

where $r \geqslant 1$. This result is a refinement of M. Hayajneh, S. Hayajneh and F. Kittaneh's result.

Mathematics subject classification (2010): 15A60, 15A18.
Keywords and phrases: Weak log-majorization, unitarily invariant norms, positive definite matrices.

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