

EXTENSIONS OF HLAWKA'S INEQUALITY FOR FOUR VECTORS

MARIUS MUNTEANU

Abstract. Given four real numbers a_1, a_2, a_3, a_4 we find necessary and sufficient conditions for the inequality

$$a_1 \sum_{i=1}^4 \|x_i\| + a_2 \sum_{1 \leq i < j \leq 4} \|x_i + x_j\| + a_3 \sum_{1 \leq i < j < k \leq 4} \|x_i + x_j + x_k\| + a_4 \left\| \sum_{i=1}^4 x_i \right\| \geq 0$$

to be satisfied for all x_1, x_2, x_3, x_4 in an inner product space, thus providing an extension of Hlawka's inequality for four vectors. As a consequence, we show that

$$\sum_{1 \leq i \leq 4} \|x_i\| - \sum_{1 \leq i < j \leq 4} \|x_i + x_j\| + \sum_{1 \leq i < j < k \leq 4} \|x_i + x_j + x_k\| + \left\| \sum_{i=1}^4 x_i \right\| \geq 0$$

and determine when equality occurs.

Mathematics subject classification (2010): 46C99, 47A30, 47A63.

Keywords and phrases: Hlawka's inequality.

REFERENCES

- [1] D. D. ADAMOVIĆ, *Généralisation d'une identité de Hlawka et de l'inegalite correspondante*, Mat. Vesnik **16** (1964), 39–43.
- [2] J. BRETAGNOLLE, D. DACUNHA-CASTELLE, AND J. KRIVINE, *Lois stables et espaces L^p* , Ann. Inst. Henri Poincaré, série B, **2** (1966), 231–259.
- [3] D. Z. DJOKOVIĆ, *Generalizations of Hlawka's inequality*, Glas. Mat.-Fiz. Astronom., Ser II, Društvo Mat. Fiz. Hrvatske, **18** (1963), 169–175.
- [4] W. FECHNER, *Hlawka's functional inequality*, Aequationes Math. **87** (2014), no. 1-2, 71–87.
- [5] C. HERTZ, *The theory of p -spaces with an application to convolution operators*, Trans. Amer. Math. Soc. **154** (1971), 69–82.
- [6] H. HORNICH, *Eine Ungleichung für Vektorlängen*, Math. Z. **48** (1942), 268–274.
- [7] J. LINDENSTRAUSS, *On the extension of operators with a finite-dimensional range*, Illinois J. Math. **8** (1964), 488–499.
- [8] D. S. MITRINOVIC, *Analytic Inequalities*, Springer-Verlag, Berlin, 1970.
- [9] L. M. KELLY, D. M. SMILEY, M. F. SMILEY, *Two dimensional spaces are quadrilateral spaces*, Am. Math. Month. **72** (1965), 753–754.
- [10] D. M. SMILEY, M. F. SMILEY, *The polygonal inequalities*, Amer. Math. Monthly **71** (1964), 755–760.
- [11] A. SIMON, P. VOLKMANN, *On Two Geometric inequalities*, Ann. Math. Sil. **9** (1995), 137–140.
- [12] D. YOST, *L_1 contains every two-dimensional normed spaces*, Ann. Polonica Math. **49** (1988), 17–19.
- [13] MATHOVERFLOW.NET, *Absolute value inequality for complex numbers*, <https://mathoverflow.net/questions/167685/absolute-value-inequality-for-complex-numbers/167769>.