

BUSEMANN-PETTY PROBLEMS FOR L_p MIXED INTERSECTION BODIES

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Abstract. The notion of L_p mixed intersection bodies was introduced by Ma. In this paper, we consider the Busemann-Petty problems for the L_p mixed intersection bodies.

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