Journal of Mathematical Inequalities
Volume 13, Number 4 (2019), 929–935
doi:10.7153/jmi-2019-13-66

THE LOGARITHMIC INTERSECTION BODY

LIJUAN LIU

Abstract. Haberl and Ludwig extended the classical intersection body to L_p space, and they showed that the classical intersection body is the limit case of the L_p intersection body. In this paper, we introduce the logarithmic intersection body and prove that it is the limit case of the normalized L_p intersection body. The affine nature of the logarithmic intersection body operator is demonstrated. Furthermore, a positive answer to the log-Busemann-Petty problem is given.

Mathematics subject classification (2010): 52A40, 53A15.
Keywords and phrases: Star body, radial function, the logarithmic intersection bodies, dual log mixed volumes.

REFERENCES

[1] G. BERCK, *Convexity of L_p-intersection bodies*, Adv. Math. 222 (2009), no. 3, 920–936.
[2] K. BÖRÖCZKY, E. LUTWAK, D. YANG, AND G. ZHANG, *The log-Brunn-Minkowski inequality*, Adv. Math. 231 (2012), no. 3-4, 1974–1997.
[3] K. BÖRÖCZKY, E. LUTWAK, D. YANG, AND G. ZHANG, *The logarithmic Minkowski problem*, J. Amer. Math. Soc. 26 (2013), no. 3, 831–852.
[4] K. BÖRÖCZKY, P. HEGEDUS, G. ZHU, *On the discrete logarithmic Minkowski problem*, Int. Math. Res. Not. (2016), no. 6, 1807–1838.
[5] S. CAMPI, *Convex intersection bodies in three and four dimensions*, Mathematika 46 (1999), no. 1, 15–27.
[6] R. J. GARDNER, *Intersection bodies and the Busemann-Petty problem*, Trans. Amer. Math. Soc. 342 (1994), no. 1, 435–445.
[7] R. J. GARDNER, *A positive answer to the Busemann-Petty problem in three dimensions*, Ann. of Math. 140 (1994), no.2, 435–447.
[8] R. J. GARDNER, *Geometric Tomography, 2nd edition*, Cambridge Univ. Press, Cambridge, 2006.
[9] R. J. GARDNER, D. HUG, W. WEIL, AND D. YE, *The dual Orlicz-Brunn-Minkowski theory*, J. Math. Anal. Appl. 430 (2015), no. 2, 810–829.
[10] R. J. GARDNER, A. KOLDOBSKY, T. SCHLUMPRECHT, *An analytic solution to the Busemann-Petty problem on sections of convex bodies*, Ann. of Math. 149 (1999), no. 2, 691–703.
[11] P. GOODEY, W. WEIL, *Intersection bodies and ellipsoids*, Mathematika 42 (1995), no. 2, 295–304.
[12] C. HABERL, *L_p intersection bodies*, Adv. Math. 217 (2008), no. 6, 2599–2624.
[13] C. HABERL, M. LUDWIG, *A characterization of L_p intersection bodies*, International Mathematics Research Notices, Art. ID 10548 (2006).
[14] N. J. KALTON, A. KOLDOBSKY, *Intersection bodies and L_p spaces*, Adv. Math. 196 (2005), 257–275.
[15] A. KOLDOBSKY, *Fourier Analysis in Convex Geometry*, American Mathematical Society Press, Providence, 2005.
[16] A. KOLDOBSKY, G. PAOURIS AND M. ZYMONOPOULOU, *Complex Intersection Bodies*, J. Lond. Math. Soc. 88 (2013), 538–562.
[17] M. LUDWIG, *Intersection bodies and valuations*, Amer. J. Math. 128 (2006), no. 6, 1409–1428.
[18] E. LUTWAK, *Intersection bodies and dual mixed volumes*, Adv. Math. 71 (1988), no. 2, 232–261.
[19] C. SAROGLOU, *Remarks on the conjectured log-Brunn-Minkowski inequality*, Geom. Dedicata 177 (2015), no. 1, 353–365.

Paper JMI-13-66

[20] R. SCHNEIDER, *Convex Bodies: The Brunn-Minkowski Theory*, Second ed., Cambridge University Press, Cambridge, 2014.
[21] A. STANCU, *The discrete planar L_0-Minkowski problem*, Adv. Math., 167 (2002), no. 1, 160–174.
[22] A. STANCU, *On the number of solutions to the discrete two dimensional L_0-Minkowski problem*, Adv. Math. 180 (2003), no. 1, 290–323.
[23] A. STANCU, *The logarithmic Minkowski inequality for non-symmetric convex bodies*, Adv. Appl. Math. 73 (2016), 43–58.
[24] W. WANG, L. LIU, *The dual log-Brunn-Minkowski inequality*, Taiwanese J. Math. 20, (2016), no. 4, 909–919.
[25] W. WANG, N. ZHANG, *The normalized L_p intersection bodies*, Math. Inequal. Appl. 21, (2018), no. 2, 353–367.
[26] J. YUAN, W. S. CHEUNG, *L_p-intersection bodies*, J. Math. Anal. Appl. 339 (2008), no. 2, 1431–1439.
[27] C. ZHAO, G. LENG, *Brunn-Minkowski inequality for mixed intersection bodies*, J. Math. Anal. Appl. 301 (2005), no. 1, 115–123.
[28] G. ZHANG, *A positive solution to the Busemann-Petty problem in $\mathbb{R}^4$*, Ann. of Math. 149 (1999), no. 2, 535–543.
[29] B. ZHU, J. ZHOU, W. XU, *Dual Orlicz-Brunn-Minkowski theory*, Adv. Math. 264 (2014), 700–725.
[30] G. ZHU, *The logarithmic Minkowski problem for polytopes*, Adv. Math. 262 (2014), 909–931.