

## ON RETARDED NONLINEAR INTEGRAL INEQUALITIES OF GRONWALL AND APPLICATIONS

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**Abstract.** By some new analysis techniques, we generalize the results presented by Pachpatte in [8] to nonlinear retarded inequalities, and also investigate some new forms. Some examples are presented to illustrate our results at the end.

**Mathematics subject classification (2010):** 26D10, 26D15, 26D20, 34A12, 34A40.

**Keywords and phrases:** Integral inequality, analysis technique, estimation, retarded integral and differential equation.

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