

COMMUTATORS OF MULTILINEAR CALDERÓN-ZYGMUND OPERATORS WITH KERNELS OF DINI'S TYPE AND APPLICATIONS

PU ZHANG AND JIE SUN

Abstract. Let T be a multilinear Calderón-Zygmund operator of type ω with $\omega(t)$ being non-decreasing and satisfying a kind of Dini's type condition. Let $T_{\Pi\bar{b}}$ be the iterated commutators of T with BMO functions. The weighted strong and weak $L(\log L)$ -type endpoint estimates for $T_{\Pi\bar{b}}$ with multiple weights are established. Some boundedness properties on weighted variable exponent Lebesgue spaces are also obtained. As applications, multiple weighted estimates for iterated commutators of bilinear pseudo-differential operators and paraproducts with mild regularity are given.

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REFERENCES

- [1] R. R. COIFMAN AND Y. MEYER, *On commutators of singular integrals and bilinear singular integrals*, Trans. Amer. Math. Soc., **212** (1975), 315–331.
- [2] R. R. COIFMAN AND Y. MEYER, *Commutateurs d'intégrales singulières et opérateurs multilinéaires*, Ann. Inst. Fourier (Grenoble), **28** (1978), no. 3, 177–202.
- [3] D. CRUZ-URIBE AND A. FIORENZA, *Variable Lebesgue Spaces: Foundations and Harmonic Analysis*, Birkhäuser/Springer, Basel (2013).
- [4] D. CRUZ-URIBE, A. FIORENZA AND C. J. NEUGEBAUER, *Weighted norm inequalities for the maximal operator on variable Lebesgue spaces*, J. Math. Anal. Appl., **304** (2012), 744–760.
- [5] D. CRUZ-URIBE AND L.-A. WANG, *Variable Hardy spaces*, Indiana Univ. Math. J., **63** (2014), no. 2, 447–493.
- [6] D. CRUZ-URIBE AND L.-A. WANG, *Extrapolation and weighted norm inequalities in the variable Lebesgue spaces*, Trans. Amer. Math. Soc., **369** (2017), no. 2, 1205–1235.
- [7] L. DIENING, P. HARJULEHTO, P. HÄSTÖ AND M. RŮŽIČKA, *Lebesgue and Sobolev Spaces with Variable Exponents*, Lecture Notes in Math. vol. 2017, Springer, Heidelberg (2011).
- [8] X. T. DUONG, L. GRAFAKOS AND L. YAN, *Multilinear operators with non-smooth kernels and commutators of singular integrals*, Trans. Amer. Math. Soc., **362** (2010), 2089–2113.
- [9] C. FEFFERMAN AND E. M. STEIN, *H^p spaces of several variables*, Acta Math., **129** (1972), 137–193.
- [10] J. GARCÍA-CUERVA AND J. L. RUBIO DE FRANCIA, *Weighted Norm Inequalities and Related Topics*, North-Holland Math. Studies, vol. 116, North-Holland Publishing Co., Amsterdam (1985).
- [11] L. GRAFAKOS, *Modern Fourier Analysis, 3rd ed.*, Grad. Texts in Math., vol. 250, Springer, New York, 2014.
- [12] L. GRAFAKOS, L. LIU, D. MALDONADO AND D. YANG, *Multilinear analysis on metric spaces*, Dissertationes Math., **497** (2014), 121 pp.
- [13] L. GRAFAKOS AND R. H. TORRES, *Multilinear Calderón-Zygmund theory*, Adv. Math., **165** (2002), 124–164.
- [14] L. GRAFAKOS AND R. H. TORRES, *Maximal operator and weighted norm inequalities for multilinear singular integrals*, Indiana Univ. Math. J., **51** (2002), 1261–1276.
- [15] C. KENIG AND E. M. STEIN, *Multilinear estimates and fractional integration*, Math. Res. Lett., **6** (1999), no. 1, 1–15.

- [16] A. K. LERNER, S. OMBROSI, C. PÉREZ, R. H. TORRES AND R. TRUJILLO-GONZÁLEZ, *New maximal functions and multiple weights for the multilinear Calderón-Zygmund theory*, Adv. Math., **220** (2009), 1222–1264.
- [17] K. LI AND W. SUN, *Weak and strong type weighted estimates for multilinear Calderón-Zygmund operators*, Adv. Math., **254** (2014), 736–771.
- [18] Z. LIU AND S. LU, *Endpoint estimates for commutators of Calderón-Zygmund type operators*, Kodai Math. J., **25** (2002), no. 1, 79–88.
- [19] G. LU AND P. ZHANG, *Multilinear Calderón-Zygmund operators with kernels of Dini's type and applications*, Nonlinear Analysis, **107** (2014), 92–117.
- [20] D. MALDONADO AND V. NAIBO, *Weighted norm inequalities for paraproducts and bilinear pseudodifferential operators with mild regularity*, J. Fourier Anal. Appl., **15** (2009), 218–261.
- [21] C. PÉREZ, *Endpoint estimates for commutators of singular operators*, J. Funct. Anal., **128** (1995), 163–185.
- [22] C. PÉREZ, G. PRADOLINI, R. H. TORRES AND R. TRUJILLO-GONZÁLEZ, *End-point estimates for iterated commutators of multilinear singular integrals*, Bull. London Math. Soc., **46** (2014), 26–42.
- [23] C. PÉREZ AND R. H. TORRES, *Minimal regularity conditions for the end-point estimate of bilinear Calderón-Zygmund operators*, Proc. Amer. Math. Soc. Series B, **1** (2014), 1–13.
- [24] M. M. RAO AND Z. D. REN, *Theory of Orlicz Spaces*, Marcel Dekker Inc., New York (1991).
- [25] E. M. STEIN, *Harmonic Analysis: Real Variable Methods, Orthogonality, and Oscillatory Integrals*, Princeton Univ. Press, Princeton, New Jersey (1993).
- [26] K. YABUTA, *Generalizations of Calderón-Zygmund operators*, Studia Math., **82** (1985), no. 1, 17–31.
- [27] P. ZHANG AND H. XU, *Sharp weighted estimates for commutators of Calderón-Zygmund type operators*, Acta Math. Sinica (Chinese Series), **48** (2005), no. 4, 625–636.