

## COMPLETE REFINEMENTS OF THE BEREZIN NUMBER INEQUALITIES

MOJTABA BAKHERAD, RAHMATOLLAH LASHKARIPOUR, MONIRE  
HAJMOHAMADI AND ULAS YAMANCI

**Abstract.** In this paper, several refinements of the Berezin number inequalities are obtained. We generalize inequalities involving powers of the Berezin number for product of two operators acting on a reproducing kernel Hilbert space  $\mathcal{H} = \mathcal{H}(\Omega)$  and also improve them. Among other inequalities, it is shown that if  $A, B \in \mathcal{B}(\mathcal{H})$  such that  $|AB| = B^*|A|$ ,  $f$  and  $g$  are nonnegative continuous functions on  $[0, \infty)$  satisfying  $f(t)g(t) = t$  ( $t \geq 0$ ), then

$$\text{ber}^p(AB) \leq r^p(B) \times \left( \text{ber} \left( \frac{1}{\alpha} f^{\alpha p}(|A|) + \frac{1}{\beta} g^{\beta p}(|A^*|) \right) - r_0 \left( \langle f^2(|A|) \hat{k}_\lambda, \hat{k}_\lambda \rangle^{\alpha p/4} - \langle g^2(|A^*|) \hat{k}_\lambda, \hat{k}_\lambda \rangle^{\beta p/4} \right)^2 \right)$$

for every  $p \geq 1, \alpha \geq \beta > 1$  with  $\frac{1}{\alpha} + \frac{1}{\beta} = 1$ ,  $\beta p \geq 2$  and  $r_0 = \min\{\frac{1}{\alpha}, \frac{1}{\beta}\}$ .

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### REFERENCES

- [1] M. W. ALOMARI, *Numerical radius inequalities for Hilbert space operators*, Math. USSR-Izv. ArX-ive:1810.05710v2.
- [2] M. BAKHERAD, *Some Berezin number inequalities for operator matrices*, Czechoslovak Math. J. **68** (4) (2018), 997–1009.
- [3] M. BAKHERAD, M. T. KARAEV, *Berezin number inequalities for operators*, Concr. Oper. **6** (2019), no. 1, 33–43.
- [4] F. A. BEREZIN, *Covariant and contravariant symbols for operators*, Math. USSR-Izv. **6** (1972), 1117–1151.
- [5] F. A. BEREZIN, *Quantizations*, Math. USSR-Izv. **8** (1974), 1109–1163.
- [6] H. BOHR, *A theorem concerning power series*, Proc. Lond. Math. Soc., **2**(13) (1914), 1–5.
- [7] M. T. GARAYEV, M. GÜRDAL, A. OKUDAN, *Hardy-Hilbert's inequality and a power inequality for Berezin numbers for operators*, Math. Inequal. Appl. **3** (2016), 883–891.
- [8] M. T. GARAYEV, M. GÜRDAL, S. SALTAN, *Hardy type inequality for reproducing kernel Hilbert space operators and related problems*, Positivity **21** (2017), 1615–1623.
- [9] M. HAJMOHAMADI, R. LASHKARIPOUR, M. BAKHERAD, *Some generalizations of numerical radius on off-diagonal part of  $2 \times 2$  operator matrices*, J. Math. Inequal. **12** (2) (2018), 447–457.
- [10] M. HAJMOHAMADI, R. LASHKARIPOUR, M. BAKHERAD, *Improvements of Berezin number inequalities*, Linear and Multilinear Algebra, <https://doi.org/10.1080/03081087.2018.1538310> (to appear).
- [11] M. HAJMOHAMADI, R. LASHKARIPOUR, M. BAKHERAD, *Further refinements of generalized numerical radius inequalities for Hilbert space operators*, Georgian Math. J. <https://doi.org/10.1515/gmj-2019-2023> (to appear).
- [12] P. R. HALMOS, *A Hilbert Space Problem Book*, 2nd ed., Springer, New York, 1982.
- [13] M. T. KARAEV, *Berezin symbol and invertibility of operators on the functional Hilbert spaces*, J. Funct. Anal. **238** (2006), 181–192.

- [14] M. T. KARAEV, *Functional analysis proofs of Abels theorems*, Proc. Amer. Math. Soc. **132** (2004), 2327–2329.
- [15] M. T. KARAEV, S. SALTAN, *Some results on Berezin symbols*, Complex Var. Theory Appl. **50** (3) (2005), 185–193.
- [16] T. KOSEM, *Inequalities between  $\|f(A+B)\|$  and  $\|f(A)+f(B)\|$* , Linear Algebra Appl. **418** (1) (2006), 153–160.
- [17] F. KITTANEH, *Notes on some inequalities for Hilbert space operators*, Publ. Res. Inst. Math. Sci. **24** (1988), 283–293.
- [18] F. KITTANEH, *Spectral radius inequalities for Hilbert space operators*, Proc. Amer. Math. Soc., **134** (2) (2005), 385–390.
- [19] F. KITTANEH, Y. MANASRAH, *Improved Young and Heinz inequalities for matrices*, J. Math. Anal. Appl. **361** (2010), 262–269.
- [20] C. A. MCCARTHY,  $C_p$ , Israel J. Math. **5** (1967), 249–271.
- [21] E. NORDGREN, P. ROSENTHAL, *Boundary values of Berezin symbols*, Oper. Theory Adv. Appl. **73** (1994), 362–368.
- [22] U. YAMANCI, M. GÜRDAL, M. T. GARAYEV, *Berezin number inequality for convex function in reproducing kernel Hilbert space*, Filomat, **31** (2017), 5711–5717.
- [23] U. YAMANCI M. GÜRDAL, *On numerical radius and Berezin number inequalities for reproducing kernel Hilbert space*, New York J. Math. **23** (2017), 1531–1537.
- [24] U. YAMANCI, M. T. GARAYEV, C. ÇELİK, *Hardy-Hilbert type inequality in reproducing kernel Hilbert space: its applications and related results*, Linear and Multilinear Algebra **67** (4) (2019), 830–842.