

SOME RESULTS FOR THE COMMUTATORS OF GENERALIZED HAUSDORFF OPERATOR

AMJAD HUSSAIN AND AMNA AJAIB

Abstract. In this paper, we study commutator of generalized Hausdorff operator on function spaces. We mainly discuss the continuity criteria for such commutator operator when the symbol functions are either from central-*BMO* or Lipschitz class of functions.

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