

AN IMPROVED RESULT OF A WEIGHTED TRIGONOMETRIC INEQUALITY IN ACUTE TRIANGLES WITH APPLICATIONS

JIAN LIU

Abstract. An improved inequality of a weighted trigonometric inequality in acute triangles is established by using the simplest arithmetic-geometric mean inequality, which also is an improvement of the well known Wolstenholme inequality for non-obtuse triangles. Its two equivalent weighted inequalities for the strengthened versions of the Erdős-Mordell inequality and Barrow's inequality are obtained. Some applications are given by new results and five relevant interesting conjectures are also put forward.

Mathematics subject classification (2010): 51M16.

Keywords and phrases: Acute triangle, non-obtuse triangle, Wolstenholme inequality, Erdős-Mordell inequality, point.

REFERENCES

- [1] O. BOTTEMA, *Geometric inequalities*, Groningen: Wolters-Noordhoff, The Netherlands, 1969.
- [2] T. O. DAO, T. D. NGUYEN, N. M. PHAM, *A strengthened version of the Erdős-Mordell inequality*, *Forum Geom.*, 16(2016), 317–321.
- [3] P. ERDŐS, *Problem 3740*, *Amer.Math.Monthly.*, 42(1935), 396.
- [4] T. HAYASHI, *Two theorems on complex numbers*, *Tôhoku Math.J.*, 4(1913–1914), 68–70.
- [5] J. LIU, *Three sine inequality*, Harbin: Harbin institute of technology press, 2018.
- [6] J. LIU, *Several trigonometric inequalities for triangles (Chinese)*, *Teaching Montly.*, 11(1994), 10–13.
- [7] J. LIU, *A Geometric inequality with one parameter for a point in the plane of a triangles*, *J. Math. Inequal.*, 8, 1(2014), 91–106.
- [8] J. LIU, *Sharpened versions of the Erdős-Mordell inequality*, *J. Inequal. Appl.*, 2015: 206(2015), 12.pp.
- [9] J. LIU, *Refinements of the Erdős-Mordell inequality, Barrow's inequality, and Oppenheim's inequality*, *J. Inequal. Appl.*, 2016: 9(2016), 18.pp.
- [10] J. LIU, *New refinements of the Erdős-Mordell inequality*, *J. Math. Inequal.*, 12, 1(2018), 63–75.
- [11] D. S. MITRINOVIĆ, J. E. PEČARIĆ, V. VOLENCE. *Recent Advances in Geometric Inequalities*, Dordrecht-Boston-London: Kluwer Academic Publishers, 1989.
- [12] B. MALESEVIĆ, M.PETROVIĆ, B.POPKONSTANTINOVIC, *On the Extension of the Erdős-Mordell type inequalities*, *Math. Inequal. Appl.*, 17, 1(2014), 269–281.
- [13] D. S. MARINESCU, M. MONEA, *About a strengthened version of the Erdős-Mordell inequality*, *Forum Geom.*, 17(2017), 197–202.
- [14] A. OPPENHEIM, *Problem E 1838*, *Amer.Math.Monthly.*, 72(1965), 1129.
- [15] N. OZEKI, *On P.Erdős' inequality for the triangle*, *J.College Arts Sci.Chiba Univ.*, 2(1957), 247–250.
- [16] A. W. WALKER, *Problem E 2388*, *Amer.Math.Monthly.*, 79(1935), 1135.
- [17] J. WOLSTENHOLME, *A Book of Mathematical Problems*, London-Cambridge, 1867.