

## INEQUALITIES FOR GENERALIZED PARAMETRIC MARCINKIEWICZ INTEGRALS ALONG POLYNOMIAL COMPOUND CURVES

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*Abstract.* Under some weaker size conditions assumed on the integral kernels both on the unit sphere and in the radial directions, the sharp  $L^p$  boundedness was proved for the generalized parametric Marcinkiewicz integrals along polynomial compound curves via an extrapolation argument. As applications, the corresponding results for generalized parametric Marcinkiewicz integral operators related to the Littlewood-Paley  $g_\lambda^*$ -functions and area integrals are also established.

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