

q -GENERALIZED BERNSTEIN–DURRMEYER POLYNOMIALS

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Abstract. The purpose of the present paper is to introduce a q -Durrmeyer variant of generalized-Bernstein operators proposed by Chen et al. (2017). The convergence rate of these operators is examined by means of the Lipschitz class and the Peetre's K -functional. Also, we define the bivariate case of these operators and study the degree of approximation with the aid of the partial moduli of continuity and higher order modulus of continuity via Peetre's K -functional approach. Finally, we introduce the GBS (Generalized Boolean Sum) of the considered operators and investigate the approximation of the Bögel continuous and Bögel differentiable functions with the aid of the Lipschitz class and the mixed modulus of smoothness. Some numerical examples with illustrative graphics have been added to validate the theoretical results and also compare the rate of convergence by using Matlab algorithms.

Mathematics subject classification (2010): 41A10, 41A25, 41A36, 41A60.

Keywords and phrases: Bögel continuous, Bögel differentiable, generalized Boolean sum, Peetre's K -functional, Lipschitz class, mixed modulus of smoothness.

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