

SOME GENERAL GRADIENT ESTIMATES FOR TWO NONLINEAR PARABOLIC EQUATIONS ALONG RICCI FLOW

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Abstract. In this paper, by maximum principle and cutoff function, we investigate gradient estimates for positive solutions to two nonlinear parabolic equations along the Ricci flow. As applications, the related Harnack inequalities for positive solutions to the nonlinear parabolic equations along the Ricci flow are derived. These results can be regarded as generalizations of the results of Li-Yau, J. Y. Li, Hamilton and Li-Xu to a more general nonlinear parabolic equation along the Ricci flow. Our results also improve the estimates of S. P. Liu, J. Sun and Y. Y. Yang to the nonlinear parabolic equation along the Ricci flow.

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