

## ON THE REVERSE HARDY-TYPE INTEGRAL INEQUALITIES IN THE WHOLE PLANE WITH THE EXTENDED RIEMANN-ZETA FUNCTION

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*Abstract.* In the present paper, using weight functions we obtain some equivalent conditions of two kinds of the reverse Hardy-type integral inequalities with a nonhomogeneous kernel in the whole plane. The constant factors, which are related to the extended Riemann zeta function, are proved to be the best possible. In the form of applications, a few equivalent conditions of two kinds of the reverse Hardy-type integral inequalities with the homogeneous kernel in the whole plane are deduced. We also consider some particular cases.

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