

## AN ELEMENTARY THREE-VARIABLE INEQUALITY WITH CONSTRAINTS FOR THE POWER FUNCTION OF THE NORMS ON SOME METRIC SPACES

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*Abstract.* A three-variable inequality with constraints for the  $p$ -power function of the norms on some metric spaces is established by an elementary technique. The new inequality performs sharper than the classical triangle inequality in the case  $p = 1$ . The performance of the involved corollaries are compared with Jensen's inequality and Clarkson's inequality.

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