

ON THE WILKER AND HUYGENS–TYPE INEQUALITIES

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Abstract. Chen and Cheung [3] established sharp Wilker and Huygens-type inequalities. These authors also proposed three conjectures on Wilker and Huygens-type inequalities. In this paper, we consider these conjectures. We also present sharp Wilker and Huygens-type inequalities.

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REFERENCES

- [1] A. BARICZ, J. SÁNDOR, *Extensions of generalized Wilker inequality to Bessel functions*, J. Math. Inequal. 2 (2008) 397–406.
- [2] C.-P. CHEN, W.-S. CHEUNG, *Inequalities and solution to Oppenheim's problem*, Integral Transforms Spec. Funct. 23 (2012), no. 5, 325–336.
- [3] C.-P. CHEN, W.-S. CHEUNG, *Sharpness of Wilker and Huygens type inequalities*, J. Inequal. Appl. 2012 (2012) 72, <http://dx.doi.org/10.1186/1029-242X-2012-72>.
- [4] C.-P. CHEN, R.B. PARIS, *Series representations of the remainders in the expansions for certain trigonometric and hyperbolic functions with applications*, <http://arxiv.org/abs/1601.03180>.
- [5] C.-P. CHEN, F. QI, *A double inequality for remainder of power series of tangent function*, Tamkang J. Math. 34 (2003), no.4, 351–355.
- [6] C.-P. CHEN, J. SÁNDOR, *Inequality chains for Wilker, Huygens and Lazarević type inequalities*, J. Math. Inequal. 8 (2014), no. 1, 55–67.
- [7] C. D'ANIELLO, *On some inequalities for the Bernoulli numbers*, Rend. Circ. Mat. Palermo 43 (1994) 329–332.
- [8] B.-N. GUO, B.-M. QIAO, F. QI, W. LI, *On new proofs of Wilker inequalities involving trigonometric functions*, Math. Inequal. Appl. 6 (2003) 19–22.
- [9] C. HUYGENS, *Oeuvres Completes 1888–1940*, Société Hollondaise des Science, Haga.
- [10] C. MORTICI, *The natural approach of Wilker-Cusa-Huygens inequalities*, Math. Inequal. Appl. 14 (2011) 535–541.
- [11] C. MORTICI, *A subtly analysis of Wilker inequality*, Appl. Math. Comput. 231 (2014) 516–520.
- [12] E. NEUMAN, *One- and two-sided inequalities for Jacobian elliptic functions and related results*, Integral Transforms Spec. Funct. 21 (2010) 399–407.
- [13] E. NEUMAN, *On Wilker and Huygens type inequalities*, Math. Inequal. Appl. 15 (2012), no. 2, 271–279.
- [14] E. NEUMAN, *Wilker and Huygens-type inequalities for the generalized trigonometric and for the generalized hyperbolic functions*, Appl. Math. Comput. 230 (2014) 211–217.
- [15] E. NEUMAN, J. SÁNDOR, *On some inequalities involving trigonometric and hyperbolic functions with emphasis on the Cusa-Huygens, Wilker and Huygens inequalities*, Math. Inequal. Appl. 13 (2010) 715–723.
- [16] I. PINELIS, *L'Hospital rules of monotonicity and Wilker-Anglesio inequality*, Amer. Math. Monthly, 111 (2004) 905–909.
- [17] J. S. SUMNER, A. A. JAGERS, M. VOWE, J. ANGLESIO, *Inequalities involving trigonometric functions*, Amer. Math. Monthly 98 (1991) 264–267.
- [18] J. B. WILKER, *Problem E 3306*, Amer. Math. Monthly 96 (1989) 55.

- [19] S.-H. WU, *On extension and refinement of Wilker's inequality*, Rocky Mountain J. Math. 39 (2009) 683–687.
- [20] S.-H. WU, A. BARICZ, *Generalizations of Mitrinović, Adamović and Lazarević's inequalities and their applications*, Publ. Math. Debrecen 75 (2009) 447–458.
- [21] S.-H. WU, H. M. SRIVASTAVA, *A weighted and exponential generalization of Wilker's inequality and its applications*, Integral Transforms and Spec. Funct. 18 (2007) 529–535.
- [22] S.-H. WU AND H. M. SRIVASTAVA, *A further refinement of Wilker's inequality*, Integral Transforms and Spec. Funct. 19 (2008) 757–765.
- [23] L. ZHANG AND L. ZHU, *A new elementary proof of Wilker's inequalities*, Math. Inequal. Appl. 11 (2008) 149–151.
- [24] J.-L. ZHAO, Q.-M. LUO, B.-N. GUO, F. QI, *Remarks on inequalities for the tangent function*, Hacet. J. Math. Stat. 41 (2012), no. 4, 499–506.
- [25] L. ZHU, *A new simple proof of Wilker's inequality*, Math. Inequal. Appl. 8 (2005) 749–750.
- [26] L. ZHU, *Some new Wilker-type inequalities for circular and hyperbolic functions*, Abstr. Appl. Anal. 2009 (2009), Article ID 485842, <http://www.hindawi.com/journals/aaa/2009/485842/>.
- [27] L. ZHU, *A source of inequalities for circular functions*, Comput. Math. Appl. 58 (2009) 1998–2004.
- [28] L. ZHU, *Inequalities for Hyperbolic functions and their Applications*, J. Inequal. Appl. 2010 (2010), Article ID 130821, <http://www.emis.de/journals/HOA/JIA/Volume2010/130821.abs.html>.