

MORREY TYPE TEICHMÜLLER SPACE AND HIGHER BERS MAPS

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Abstract. In this paper, we focus on the set of univalent analytic functions f with $\log f' \in H_K^2$. Motivated by the study of BMO-Teichmüller spaces and Morrey type spaces, we establish several equivalent characterizations of Morrey type domains. Furthermore, we show that the higher Bers maps, induced by the higher Schwarzian differential operators, are holomorphic in Morrey type Teichmüller spaces. Finally, one of connected components in the small pre-logarithmic derivative model of the Morrey type Teichmüller space is also obtained.

Mathematics subject classification (2010): 30C20, 30C62, 30D99.

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