

SHARP COEFFICIENTS BOUNDS FOR CLASS OF ALMOST STARLIKE MAPPINGS OF ORDER α IN \mathbb{C}^n

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Abstract. Let Ω be the bounded starlike circular domain. In this paper, we obtain the sharp bounds for the Fekete-Szegő functional $|A_3 - \mu A_2^2|$ of the class $\mathcal{AS}_\alpha^*(\Omega)$ of almost starlike mappings of order α in \mathbb{C}^n ($n \geq 2$), where $\mu \in \mathbb{R}$, and A_2, A_3 are the first two coefficients of the homogeneous expansion of mappings $f \in \mathcal{AS}_\alpha^*(\Omega)$. Our results can be regarded as the extensions of corresponding works from the case in one dimension to the case in higher dimensions.

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