

A PRIORI AND POSTERIORI ERROR ESTIMATES OF LEGENDRE GALERKIN SPECTRAL METHODS FOR GENERAL ELLIPTIC OPTIMAL CONTROL PROBLEMS

ZULIANG LU, FEI HUANG, LI LIN, FEI CAI AND YIN YANG

Abstract. In this paper, the Legendre Galerkin spectral method is applied to solve the constrained optimal control problems governed by general elliptic equations. Under some reasonable assumptions, by using the orthogonal projection operator, we derive a priori error estimates for the spectral approximation of optimal control problems. Then, we obtain a posteriori error estimates for both the state and the control approximation, where we use the L^2 -norm for estimating the control approximation error, and the H^1 -norm or L^2 -norm for the state and co-state approximation error. Finally, some numerical experiments are presented to test our theoretical results.

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