

EQUIVALENT STATEMENTS OF A HILBERT-TYPE INTEGRAL INEQUALITY WITH THE EXTENDED HURWITZ ZETA FUNCTION IN THE WHOLE PLANE

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Abstract. By using the way of real analysis and the weight functions, a few equivalent statements of a Hilbert-type integral inequality with the nonhomogeneous kernel in the whole plane are obtained. The constant factor related the extended Hurwitz zeta function is proved to be the best possible. As applications, a few equivalent statements of a Hilbert-type integral inequality with the homogeneous kernel in the whole plane are deduced. We also consider the operator expressions and some corollaries.

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