

COMPOSITION OPERATORS AND CLOSURES OF DIRICHLET TYPE SPACES IN LOGARITHMIC BLOCH-TYPE SPACES

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Abstract. Closures of Dirichlet type spaces in logarithmic Bloch-type spaces are investigated in the paper. Moreover, the boundedness and compactness of composition operators from logarithmic Bloch-type spaces to closures of Dirichlet type spaces in logarithmic Bloch-type spaces are characterized.

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