

## ALGORITHMS FOR SPLIT COMMON NULL POINT PROBLEM WITHOUT PRE-EXISTING ESTIMATION OF OPERATOR NORM

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*Abstract.* The purpose of this paper is to present iterative methods to solve a split common null point problem in real Hilbert spaces such that the implementation of proposed iterative schemes do not require any pre-existing estimation of the norm of bounded linear operator. We give the weak and strong convergence of the proposed algorithms under some mild and standard assumptions in Hilbert spaces. A numerical example is also constructed to illustrate the algorithm for strong convergence.

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