

ONE DIMENSIONAL WEIGHTED HARDY'S INEQUALITIES AND APPLICATION

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Abstract. Let Ω be a C^2 class bounded domain of \mathbb{R}^n ($n \geq 1$). In the present paper we shall improve one dimensional weighted Hardy inequalities with one-sided boundary condition by adding sharp remainders. As an application, we shall establish n dimensional weighted Hardy inequalities with weight functions being powers of the distance function $\delta(x)$ to the boundary $\partial\Omega$. Our results will be applicable to variational problems in a coming paper [3].

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