

BEST BOUNDS FOR THE LAMBERT W FUNCTIONS

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Abstract. This paper is devoted to provide tractable closed-form upper and lower bounds for the two real branches of the Lambert W function $W(z(t))$ for all positive real variable t where $z(t)$ is increasing function on $(0, \infty)$ and bounded by zero and $-e^{-1}$.

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