

HERMITE–HADAMARD TYPE INEQUALITIES FOR OPERATOR (p, h) –CONVEX FUNCTIONS

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Abstract. Motivated by the recent work on convex functions and operator convex functions, we investigate the Hermite–Hadamard inequalities for operator (p, h) -convex functions. We also present the estimates of both sides of the Hermite–Hadamard type inequality for operator (p, h) -convex functions, where h is a non-negative function with $h(t) + h(1-t) \leq \kappa$ (κ is a positive constant) for $t \in (0, 1)$. The results are new even for the commutative case. Applications for particular cases of these inequalities are also provided.

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