

## ON A MORE ACCURATE REVERSE HILBERT-TYPE INEQUALITY IN THE WHOLE PLANE

MICHAEL TH. RASSIAS, BICHENG YANG AND ANDREI RAIGORODSKII

*Abstract.* In the present paper, using weight coefficients and applying Hermite-Hadamard's inequality, we derive a new, more accurate reverse Hilbert-type inequality in the whole plane with multi-parameters involving the cosine and natural logarithm functions. The corresponding constant factor is proved to be the best possible. We additionally consider some equivalent forms and a few particular inequalities. As an application, the obtained results are compared with some previously known results and we show that these new results are more accurate than the earlier ones.

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