

FRACTIONAL INTEGRAL OPERATORS WITH HOMOGENEOUS KERNELS ON GENERALIZED LORENTZ–MORREY SPACES

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Abstract. We establish the mapping properties of the fractional integral operators with homogeneous kernels on generalized Lorentz–Morrey spaces.

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