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NEW INEQUALITIES FOR INTERPOLATIONAL OPERATOR MEANS

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Abstract. The main goal of this article is to present several refinements and reverses of well known operator inequalities. These inequalities include operator means, operator monotone functions, operator log-convex functions and positive linear maps.

Among many other results, we show that for any $0 \leqslant \alpha, \beta \leqslant 1$,

$$f(A\nabla_{\alpha}B) \leqslant f\left((A\nabla_{\alpha}B)\nabla_{\beta}A\right)\sharp_{\alpha}f\left((A\nabla_{\alpha}B)\nabla_{\beta}B\right) \leqslant f(A)\sharp_{\alpha}f(B)$$

whenever f is a non-negative operator log-convex function, $A,B \in \mathscr{B}(\mathscr{H})$ are positive operators, and $0 \leqslant \alpha, \beta \leqslant 1$. Further, we consider some inequalities of Ando's type, and prove that if Φ is a positive linear map, then

$$\Phi(A\sharp_{\alpha}B) \leqslant \Phi\left((A\sharp_{\alpha}B)\sharp_{\beta}A\right)\sharp_{\alpha}\Phi\left((A\sharp_{\alpha}B)\sharp_{\beta}B\right) \leqslant \Phi(A)\sharp_{\alpha}\Phi(B).$$

Many other refinements and reverses are shown by invoking ideas related to the so called interpolational operator means.

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