

## SOME SIMPSON TYPE FRACTIONAL INTEGRAL INEQUALITIES FOR FUNCTIONS OF BOUNDED VARIATION

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*Abstract.* In this paper, we focus on the Simpson type fractional integral inequalities. The approximate schemes of these integral inequalities are derived as well.

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