

SOME NEW GENERALIZATIONS OF WEIGHTED DYNAMIC HARDY–KNOPP TYPE INEQUALITIES WITH KERNELS

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Abstract. In this paper, we establish some new generalizations of weighted dynamic Hardy–Knopp type inequalities with kernels on time scales and also, some new characterizations of the weights for these inequalities in different spaces. The main results will be proved by using the Hölder inequality, the Jensen inequality and the Minkowski inequality. These inequalities (when $\mathbb{T} = \mathbb{R}$) contain the characterization of Kaijser, Nikolova, Persson and Wedestig. Also, (when $\mathbb{T} = \mathbb{N}$) our results are essentially new.

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