Journal of
Mathematical
Inequalities
Volume 15, Number 4 (2021), 1613–1636 doi:10.7153/jmi-2021-15-111

MAPPING PROPERTIES OF MAXIMAL OPERATORS ON INFINITE CONNECTED GRAPHS

XIAO ZHANG, FENG LIU* AND HUIYUN ZHANG

Abstract. In this paper, we introduce Morrey, Hölder, Lipschitz and Campanato spaces on infinite connected graphs. We establish the boundedness of the Hardy-Littlewood maximal operator and its fractional variants on the above function spaces under certain conditions on graphs. The relations between Hölder spaces and Campanato spaces will be also investigated.

Mathematics subject classification (2020): Primary 42B25, 05C12; Secondary 46E35, 05C75.
Keywords and phrases: Hardy-Littlewood maximal operator and its fractional variant, infinite graph, Morrey space, Hölder space, Lipschitz space, Campanato space.

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