

DETERMINANT FOR POSITIVE OPERATORS AND OPPENHEIM'S INEQUALITY

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Abstract. In this paper, by virtue of the Specht ratio, we show Oppenheim type inequalities for the normalized determinant of positive invertible operators on a Hilbert space, and we moreover discuss Hadamard type inequalities for positive definite matrices.

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