

ON A MORE ACCURATE HILBERT-TYPE INEQUALITY INVOLVING THE PARTIAL SUMS

BING HE, YANRU ZHONG* AND BICHENG YANG

Abstract. By means of the weight coefficients, Hermite-Hadamard's inequality, the Euler-Maclaurin summation formula and Abel's summation by parts formula, a more accurate Hilbert-type inequality with the partial sums is given. The equivalent conditions of the best possible constant factor related to several parameters and some particular inequalities are also obtained.

Mathematics subject classification (2020): 26D15, 47A05.

Keywords and phrases: Weight coefficient, Euler-Maclaurin summation formula, Hilbert-type inequality, parameter, partial sum.

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