

NEW GENERALIZED RIEMANN—LIOUVILLE FRACTIONAL INTEGRAL INEQUALITIES FOR CONVEX FUNCTIONS

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Abstract. In the literature, the right-side of Hermite–Hadamard’s inequality is called trapezoid type inequality. In this paper, we obtain new integral inequalities of trapezoid type for convex functions involving generalized Riemann–Liouville fractional integrals (ψ -Riemann–Liouville fractional integrals). Our obtained inequalities generalize some recent classical integral inequalities and Riemann–Liouville fractional integral inequalities which are established in earlier works.

Mathematics subject classification (2020): 26D07, 26D15, 26D10, 26A33.

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