

MONOTONE ITERATIVE TECHNIQUE FOR S -ASYMPTOTICALLY PERIODIC PROBLEM OF FRACTIONAL EVOLUTION EQUATION WITH FINITE DELAY IN ORDERED BANACH SPACE

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Abstract. In this paper, we devote to considering S -asymptotically periodic problem of fractional evolution equation with delay in ordered Banach space. Under some weaker assumptions, we construct monotone iterative method in the presence of the lower and upper solutions to the delayed fractional evolution equation, and obtain the existence of maximal and minimal S -asymptotically periodic mild solutions. Finally, we present two examples to illustrate the feasibility of our abstract results.

Mathematics subject classification (2020): 34K30, 34K37, 35R11, 47H07, 47H08.

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