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ADDITIVE DOUBLE ρ–FUNCTIONAL INEQUALITIES IN β–HOMOGENEOUS F–SPACES

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Abstract. In this paper, we introduce and solve the following additive double ρ-functional inequalities

$$\begin{aligned}\|f(x+y+z)+f(x-y)-f(z)-2f(x)\| \leqslant &\|\rho_1(f(x+y+z)-f(x)-f(y)-f(z))\| \\ &+\|\rho_2(f(x+y+z)-f(x+y)-f(z))\|\end{aligned} \tag{1}$$

where ρ_1, ρ_2 are fixed nonzero complex numbers with $|2\rho_1|^{\beta_2}+|\rho_2|^{\beta_2}<1$, and

$$\begin{aligned}\|f(x+y+z)-f(x)-f(y)-f(z)\| \leqslant &\|\rho_1(f(x+y+z)+f(x-y)-f(z)-2f(x))\| \\ &+\|\rho_2(f(x+y+z)-f(x+y)-f(z))\|\end{aligned} \tag{2}$$

where ρ_1, ρ_2 are fixed nonzero complex numbers with $|\rho_1|^{\beta_2}+|\rho_2|^{\beta_2}<1$.

By adopting the direct method, we have made an attempt to prove the Hyers-Ulam stability of the additive double ρ-functional inequalities in β-homogeneous F-spaces.

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