

MONOTONICITY, CONVEXITY AND BOUNDS INVOLVING THE BETA AND RAMANUJAN R -FUNCTIONS

TI-REN HUANG, LU CHEN, SHEN-YANG TAN AND YU-MING CHU*

Abstract. In the article, we provide several new asymptotical sharp bounds for the functions involving the Beta function and Ramanujan R -functions via the monotonicity and convexity properties of certain combinations defined in terms of polynomials, Beta and Ramanujan R -functions.

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