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OPTIMAL BOUNDS FOR THE SÁNDOR MEAN IN TERMS OF THE COMBINATION OF GEOMETRIC AND ARITHMETIC MEANS

WEI-MAO QIAN, CHUN-LIN MA AND HUI-ZUO XU*

Abstract. In this paper, we prove that $\lambda = 1/2 - \sqrt{1-e^{-2/p}}/2$ and $\mu = 1/2 - \sqrt{6p}/(6p)$ are the best possible parameters on the interval $(0,1/2)$ such that the double inequalities

$$G^p\left[\lambda a+(1-\lambda)b, \lambda b+(1-\lambda)a\right]A^{1-p}(a,b) < X(a,b)$$
$$< G^p\left[\mu a+(1-\mu)b, \mu b+(1-\mu)a\right]A^{1-p}(a,b)$$

hold for all $p \in [1,\infty)$ and $a,b>0$ with $a \neq b$, where $G(a,b)$ is the geometric mean, $A(a,b)$ is the arithmetic mean, and $X(a,b)$ is the Sándor mean.

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