

VOLUMES OF SUB-LEVEL SETS AND THE DECAY OF OSCILLATORY INTEGRALS

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Abstract. In this paper, we present estimates for the volumes of sub-level sets and the decay of oscillatory integrals with phase functions in a family of functions definable in an o-minimal structure with compact domains.

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