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STABLE FUNCTIONS OF JANOWSKI TYPE

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Abstract. A function $f \in \mathscr{A}_1$ is said to be stable with respect to $g \in \mathscr{A}_1$ if

$$\frac{s_n(f(z))}{f(z)} \prec \frac{1}{g(z)}, \qquad z \in \mathbb{D},$$

holds for all $n \in \mathbb{N}$ where $\mathscr{A}_1$ denote the class of analytic functions f in the unit disk $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ normalized by $f(0) = 1$. Here $s_n(f(z))$, the n^{th} partial sum of $f(z) = \sum_{k=0}^{\infty} a_k z^k$ is given by $s_n(f(z)) = \sum_{k=0}^{n} a_k z^k$, $n \in \mathbb{N} \cup \{0\}$. In this work, we consider the following function

$$v_\lambda(A,B,z) = \left(\frac{1+Az}{1+Bz}\right)^\lambda$$

for $-1 \leqslant B < A \leqslant 1$ and $0 \leqslant \lambda \leqslant 1$ for our investigation. The main purpose of this paper is to prove that $v_\lambda(A,B,z)$ is stable with respect to $v_\lambda(0,B,z) = \dfrac{1}{(1+Bz)^\lambda}$ for $0 < \lambda \leqslant 1$ and $-1 \leqslant B < A \leqslant 0$. Further, we prove that $v_\lambda(A,B,z)$ is not stable with respect to itself, when $0 < \lambda \leqslant 1$ and $-1 \leqslant B < A < 0$.

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