

SECOND ORDER NONLINEAR EVOLUTIONARY SYSTEMS DRIVEN BY GENERALIZED MIXED VARIATIONAL INEQUALITIES

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Abstract. In this paper, we deal with the system formulated by abstract second order nonlinear evolution differential equations which are subject to a generalized mixed variational inequalities. Firstly, based on Ky Fan inequality theorem, we examine that the solution set of variational inequalities is bounded, closed and convex by getting rid of the rigid restriction of monotonicity. Afterwards, the existence of solutions for a class of nonlinear differential equation is discussed.

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