

HIGHER DIMENSIONS OPIAL DIAMOND- α INEQUALITIES ON TIME SCALES

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Abstract. In this paper, we generalize Opial inequality to higher dimensions on time scales. We prove the Opial Delta-nabla inequality of n variables, and then give two diamond-alpha dynamic inequalities of Opial type of n variables. As well, we introduce some special cases.

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