

NUMERICAL SEMIGROUPS BOUNDED BY A CYCLIC MONOID

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Abstract. We will say that a numerical semigroup S is bounded by a cyclic monoid if there exist integer numbers $0 \leq \alpha < \beta$ such that $S = \{x \in \mathbb{N} \mid k\alpha < x < k\beta \text{ for some } k \in \mathbb{N}\} \cup \{0\}$. The goal of this work is to study this kind of numerical semigroups. In particular, we will determine important invariants of them such as multiplicity, embedding dimension, Frobenius number and genus.

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