

## COMPOSITE CONVEX FUNCTIONS

MOHAMMAD SABABHEH, SHIGERU FURUICHI AND HAMID REZA MORADI

*Abstract.* Convex functions have played a major role in the field of Mathematical inequalities. In this paper, we introduce a new concept related to convexity, which proves better estimates when the function is somehow more convex than another.

In particular, we define what we called  $g$ -composite convexity as a generalization of log-convexity. Then we prove that  $g$ -composite convex functions have better estimates in certain known inequalities like the Hermite-Hadamard inequality, super additivity of convex functions, the Majorization inequality and some means inequalities.

Strongly related to this, we define the index of convexity as a measure of “how much the function is convex”.

Applications including Hilbert space operators, matrices and entropies will be presented in the end.

*Mathematics subject classification (2020):* Primary 26A51; Secondary 47A30, 39B62, 26D07, 47B15, 15A60.

*Keywords and phrases:* Convex functions, Jensen’s inequality, norm inequality, weak majorization.

### REFERENCES

- [1] T. ANDO AND F. HIAI, *Operator log-convex functions and operator means*, Math. Ann. **350** (3) (2011), 611–630.
- [2] S. S. DRAGOMIR, *Inequality of Hermite-Hadamard type of composite convex functions*, Frontiers in Functional Equations and Analytic Inequalities, Springer, Cham, 2019, 559–584.
- [3] S. FURUICHI, K. YANAGI AND K. KURIYAMA, *On bounds for symmetric divergence measures*, AIP Conf. Proc. **1853** (2017), 080002.
- [4] F. HANSEN, *An operator inequality*, Math. Ann. **246** (1980), 249–250.
- [5] T. KOSEM, *Inequalities between  $\|f(A+B)\|$  and  $\|f(A)+f(B)\|$* , Linear Algebra Appl. **418** (2006), 153–160.
- [6] J. MATKOWSKI, *A functional inequality characterizing convex functions, conjugacy and generalization of Holder’s and Minkowski’s inequalities*, Aequat. Math. **40** (1990), 168–180.
- [7] J. MIĆIĆ, Z. PAVIĆ AND J. PEČARIĆ, *Jensen type inequalities on quasi-arithmetic operator mean*, Scientiae Mathematicae Japonicae **73** (2011), 183–192.
- [8] H. R. MORADI AND M. SABABHEH, *Eigenvalue inequalities for  $n$ -tuple of matrices*, Linear Multilinear Algebra, <https://doi.org/10.1080/03081087.2019.1664384>.
- [9] M. A. NIELSEN AND I. L. CHUANG, *Quantum computation and quantum information*, Cambridge University Press, 2000.
- [10] M. OHYA AND D. PETZ, *Quantum entropy and its use*, Springer-Verlag, Second Edition, 2004.
- [11] H. L. PEDERSEN AND M. UCHIYAMA, *Inverses of operator convex functions, in Ordered structures and applications*, Papers from Positivity VII, ed. M. de Jeu, B. de Pagter, O. van Gaans and M. Veraar, pp. 363–370, Birkhäuser/Springer, 2016.
- [12] D. PETZ, *Quantum information theory and quantum statistics*, Springer, 2004.
- [13] M. SABABHEH, H. R. MORADI AND S. FURUICHI, *Integrals refining convex inequalities*, Bull. Malays. Math. Sci. Soc. **43** (2020), 2817–2833.
- [14] M. SABABHEHA AND H. R. MORADI, *Radical convex functions*, Medit. J. Math. **18**, 137 (2021), <https://doi.org/10.1007/s00009-021-01784-8>.

- [15] M. SABABHEH, *Log and Harmonically log-convex functions related to matrix norms*, Oper. Matrices. **10** (2) (2016), 453–465.
- [16] H. UMEGAKI, *Conditional expectation in an operator algebra, IV (entropy and information)*, Kodai Math. Sem. Rep. **14** (1962), 59–85.
- [17] J. VON NEUMANN, *Thermodynamik quantenmechanischer Gesamtheiten*, Nachr. Ges. Wiss. Göttingen, (1927), 273–291.