

## SHARP ESTIMATES FOR $m$ -LINEAR $p$ -ADIC HARDY AND HARDY-LITTLEWOOD-PÓLYA OPERATORS ON $p$ -ADIC CENTRAL MORREY SPACES

YANGKENDI DENG, DUNYAN YAN AND MINGQUAN WEI\*

*Abstract.* In this paper, we consider the mapping property of both  $m$ -linear  $p$ -adic Hardy operator  $\mathcal{H}_m^p$  and Hardy-Littlewood-Pólya operators  $T_m^p$  on  $p$ -adic central Morrey-type spaces  $B^{p_1, \lambda_1}(\mathbb{Q}_p^n) \times \cdots \times B^{p_m, \lambda_m}(\mathbb{Q}_p^n)$ . The obtained bounds turn to be sharp when  $\lambda_1 p_1 = \cdots = \lambda_m p_m$ .

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