

SOME FURTHER PROPERTIES OF DISCRETE MUCKENHOUP AND GEHRING WEIGHTS

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Abstract. The main objective of this paper is a further study of discrete Muckenhoupt and Gehring weights. We first restate monotonicity properties of Muckenhoupt and Gehring classes in terms of the corresponding norms. In addition, we establish some norm bounds for Muckenhoupt and Gehring weights. Next, we give a simple characterization of the weight belonging to both Muckenhoupt and Gehring class. Finally, we show that the transition functions, arising from inclusion problems between Muckenhoupt and Gehring classes, are decreasing. As an application, some particular examples of Muckenhoupt and Gehring power weights are also considered.

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