THE LAI LAW FOR WEIGHTED SUMS

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Abstract. Let $\{X, X_n, n \geqslant 1\}$ be a sequence of independent and identically distributed random variables with partial sums $S_n = \sum_{k=1}^{n} X_k$, $n \geqslant 1$. Lai law states that

$$\sum_{n=1}^{\infty} n^{r-2} P\{|S_n| > \varepsilon \sqrt{2n \log n}\} \begin{cases} < \infty, & \text{if } \varepsilon > \sqrt{r-1}, \\ = \infty, & \text{if } \varepsilon < \sqrt{r-1} \end{cases}$$

if and only if $EX = 0$, $EX^2 = 1$ and $E(X^2 / \log |X|)^r < \infty$, where $r > 1$. The paper will extend the result to the weighted sums under some conditions both on the weights and the moment.

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