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INEQUALITIES FOR GENERALIZED MATRIX FUNCTION AND INNER PRODUCT

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Abstract. We present inequalities related to generalized matrix function for positive semidefinite block matrices. We introduce partial generalized matrix functions corresponding to partial traces, and then provide a unified extension of the recent inequalities due to Lin [14], Zhang et al. [19, 4] and Choi [5]. Moreover, we demonstrate the application of a positive semidefinite 3×3 block matrix, which motivates us to give alternative proofs of Dragomir's inequality and Krein's inequality.

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