

A UNIFIED FRAMEWORK FOR SOLVING GENERALIZED VARIATIONAL INEQUALITIES

WENLONG SUN*, GANG LU, YUANFENG JIN* AND CHOONKIL PARK

Abstract. In this paper, a generalized variational inequality and fixed points problem is presented. An iterative algorithm is introduced for finding a solution of the generalized variational inequalities and fixed point of two uniformly Lipschitzian asymptotically quasi-pseudo-contractive operators under a nonlinear transformation. Strong convergence of the suggested algorithm is demonstrated.

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