

## SOME UNIQUENESS PROBLEMS CONCERNING MEROMORPHIC FUNCTIONS

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**Abstract.** In this paper, we investigate some uniqueness problems for forward differences  $\Delta_{\eta}^n f(z)$  and  $\Delta_{\eta} f(z)$  of a meromorphic function  $f(z)$ . When  $f(z)$  is an entire function with a Borel exceptional small function  $a(z)$ , we obtain the concrete expression of  $f(z)$  under the condition that  $\Delta_{\eta}^n f(z)$  and  $\Delta_{\eta} f(z)$  share a small function  $b(z)$  CM. When  $f(z)$  is a meromorphic function with a small deficient function  $a(z)$ , we obtain the relationship between  $\Delta_{\eta}^n f(z)$  and  $\Delta_{\eta} f(z)$  who share a small function  $b(z)$  and  $\infty$  CM.

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